MATHEMATICS OF FINANCE

WHY IS INTEREST PAID ?

1. ***Time value of money :***

Time value of money means that the value of a unity of money is different in different time periods.

The sum of money received in future is less valuable than it is today.

In other words the present worth of money received after some time will be less than money received today.

2. ***Opportunity Cost:*** The lender has a choice between using his money in different investments. If he chooses one he forgoes the return from all others. In other words lending incurs an opportunity cost due to the possible alternative uses of the lent mone

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3. ***Inflation:*** Most economies generally exhibit inflation. Inflation is a fall in the purchasing power of money. Due to inflation a given amount of money buys fewer goods in the future than it will now. The borrower needs to compensate the lender for this.

WHY IS INTEREST PAID ?

4. ***Liquidity Preference:*** **People prefer to have their resources available in a form that can immediately be converted into cash rather than a form that takes time or money to realize.**

5. ***Risk Factor:*** **There is always a risk that the borrower will go bankrupt or otherwise default on the loan. Risk is a determinable factor in fixing rate of interest.**

- **A lender generally charges more interest rate (risk premium) for taking more risk.**

TERMS

INTEREST

- **Interest is the price paid by a borrower for the use of a lender's money. If you borrow (or lend) some money from (or to) a person for a particular period you would pay (or receive) more money than your initial borrowing (or lending). This excess money paid (or received) is called *interest*.**

- *Suppose you borrow (or lend) ₹ 50,000 for a year and you pay (or receive) ₹ 55,000 after one year the difference between initial borrowing (or lending) ₹ 50,000 and end payment (or receipts) ₹ 55,000 i.e. ₹ 5,000 is the amount of interest you paid (or earned).*

TERMS

PRINCIPAL

- **Principal is initial value of lending (or borrowing).**
- **If you invest your money the value of initial investment is also called principal.**
- *Suppose you borrow (or lend) ₹ 50,000 from a person for one year. ₹ 50,000 in this example is the 'Principal.'*

  Take another example suppose you deposit ₹ 20,000 in your bank account for one year. In this example ₹ 20,000 is the principal.

TERMS

RATE OF INTEREST

- The rate at which the interest is charged for a defined length of time for use of principal generally on a yearly basis is known to be ***the rate of interest***.
- Rate of interest is usually as expressed as percentages.
- Per annum means for a year.

TERMS

ACCUMULATED BALANCE

- **Accumulated amount is the final value of an investment.**
- **It is the sum total of principal and interest earned.**
- *Suppose you deposit ₹ 50,000 in your bank for one year with an interest rate of 5% p.a. you would earn interest of ₹ 2,500 after one year. (method of computing interest will be illustrated later). After one year you will get ₹ 52,500 (principal + interest), ₹ 52,500 is accumulated amount here.*
- **Amount is also known as the balance.**

SIMPLE INTEREST

- Simple interest is the interest computed on the principal for the entire period of borrowing .
- It is calculated on the principal amount only and not on interest previously earned.
- Value of interest remains constant for each year.

$$S\,I = \frac{P.r.t}{100}$$

P = principal value

r = rate of interest per annum

t = time period in years

SIMPLE INTEREST

- $A = P + SI$

$$= P + \frac{P.r.t}{100}$$

$$= P\left(1 + \frac{rt}{100}\right)$$

Simple interest can be computed by applying following formulas:

I = Pit

A = P + I
= P + Pit
= P(1 + it)

I = A – P

Here,

A = Accumulated amount (final value of an investment)
P = Principal (initial value of an investment)
i = Annual interest rate in decimal.
I = Amount of Interest
t = Time in years

Example 1: How much interest will be earned on ₹ 2000 at 6% simple interest for 2 years?

Example 2: Sania deposited ₹ 50,000 in a bank for two years with the interest rate of 5.5% p.a. How much interest would she earn?

Example 3: In example 2 what will be the final value of investment?

Example 4: Sachin deposited ₹ 1,00,000 in his bank for 2 years at simple interest rate of 6%. How much interest would he earn? How much would be the final value of deposit?

Example 5: Find the rate of interest if the amount owed after 6 months is ₹ 1050, borrowed amount being ₹ 1000.

Example 6: Rahul invested ₹ 70,000 in a bank at the rate of 6.5% p.a. simple interest rate.

He received ₹ 85,925 after the end of term. Find out the period for which sum was invested by Rahul.

Example 7 : Kapil deposited some amount in a bank for 7 1/2 years at the rate of 6% p.a. simple interest. Kapil received ₹ 1,01,500 at the end of the term. Compute initial deposit of Kapil.

Example 8: A sum of ₹ 46,875 was lent out at simple interest and at the end of 1 year 8 months the total amount was ₹ 50,000. Find the rate of interest percent per annum.

Example 9: What sum of money will produce ₹ 28,600 as an interest in 3 years and 3 months at 2.5% p.a. simple interest?

Example 10: In what time will ₹ 85,000 amount to ₹ 1,57,675 at 4.5 % p.a. ?

EXERCISE- Set (A)

Que. 1 S.I on ₹ 3,500 for 3 years at 12% p.a. is

(a) ₹ 1,200

(b) ₹ 1,260

(c) ₹ 2,260

(d) none of these

Que. 2 P = 5,000, R = 15, T = 4 1/2 using I = PRT/100, I will be

(a) ₹ 3,375

(b) ₹ 3,300

(c) ₹ 3,735

(d) None of these

Que. 3 If P = 5,000, T = 1, I = ₹ 300, R will be

(a) 5%

(b) 4%

(c) 6%

(d) None of these

Que. 4 If P = ₹ 4,500, A = ₹ 7,200, than Simple interest i.e. I will be

(a) ₹ 2,000

(b) ₹ 3,000

(c) ₹ 2,500

(d) ₹ 2,700

Que. 5 P = ₹ 12,000, A = ₹ 16,500, T = 2 1/2 years. Rate percent per annum simple interest will be

(a) 15%

(b) 12%

(c) 10%

(d) None of these

Que. 6 P = ₹ 10,000, I = ₹ 2,500, R = 12 1/2% SI. The number of years T will be

(a) 1 1/2 years

(b) 2 years

(c) 3 years

(d) None of these

Que. 7 P = ₹ 8,500, A = ₹ 10,200, R = 12 1/2 % SI, t will be.

(a) 1 yr. 7 mth.

(b) 2 yrs.

(c) 1 1/2 yr.

(d) None of these

Que. 8 The sum required to earn a monthly interest of ₹ 1,200 at 18% per annum SI is

(a) ₹ 50,000

(b) ₹ 60,000

(c) ₹ 80,000

(d) None of these

Que. 9 A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of interest are

(a) ₹ 3,800, 31.58%

(b) ₹ 3,000, 20%

(c) ₹ 3,500, 15%

(d) None of these

Que. 10 A sum of money doubles itself in 10 years. The number of years it would triple itself is

(a) 25 years.

(b) 15 years.

(c) 20 years

(d) None of these

COMPOUND INTEREST

- We can define the compound interest as the interest that accrues when earnings for each specified period of time added to the principal thus increasing the principal base on which subsequent interest is computed.

COMPOUND INTEREST

Example 11 : Saina deposited ₹ 1,00,000 in a nationalized bank for three years. If the rate of interest is 7% p.a., calculate the interest that bank has to pay to Saina after three years if interest is compounded annually. Also calculate the amount at the end of third year.

COMPOUND INTEREST

NOTE

- In practice it is not necessary that the interest be compounded annually. For example in banks the interest is often compounded twice a year (half yearly or semi annually) i.e. interest is calculated and added to the principal after every six months. In some financial institutions interest is compounded quarterly i.e. four times a year.
- The period at the end of which the interest is compounded is called conversion period.

COMPOUND INTEREST

- The period at the end of which the interest is compounded is called conversion period.

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

COMPOUND INTEREST

$$A_n = P(1+i)^n$$

$$\text{where, } i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}}$$

$$\text{Interest} = A_n - P = P(1+i)^n - P$$

$$= P\left[(1+i)^n - 1\right]$$

n is total conversions i.e. t x no. of conversions per year

COMPOUND INTEREST

Example 12: ₹ 2,000 is invested at annual rate of interest of 10%. What is the amount after two years if compounding is done (a) Annually (b) Semi-annually (c) Quarterly (d) monthly.

COMPOUND INTEREST

Example 13 : Determine the compound amount and compound interest on ₹ 1000 at 6% compounded semi-annually for 6 years. Given that $(1 + i)^n = 1.42576$ for i = 3% and n = 12.

COMPOUND INTEREST

Example 14: Compute the compound interest on ₹ 4,000 for 1 1/2 years at 10% per annum compounded half- yearly.

COMPOUND INTEREST

Example 15: On what sum will the compound interest at 5% per annum for two years compounded annually be ₹ 1,640?

COMPOUND INTEREST

Example 16: What annual rate of interest compounded annually doubles an investment in 7 years? Given that $2^{\frac{1}{7}} = 1.104090$

COMPOUND INTEREST

Example 17: In what time will ₹ 8,000 amount to ₹ 8,820 at 10% per annum interest compounded half-yearly?

COMPOUND INTEREST

Example 18: Find the rate percent per annum if ₹ 2,00,000 amount to ₹ 2,31,525 in 1 1/2 year interest being compounded half-yearly.

COMPOUND INTEREST

Example 19: A certain sum invested at 4% per annum compounded semi-annually amounts to ₹ 78,030 at the end of one year. Find the sum.

COMPOUND INTEREST

Example 20: ₹ 16,000 invested at 10% p.a. compounded semi-annually amounts to ₹ 18,522.

Find the time period of investment.

COMPOUND INTEREST

Example 21: A person opened an account on April, 2011 with a deposit of ₹ 800. The account paid 6% interest compounded quarterly. On October 1 2011 he closed the account and added enough additional money to invest in a 6 month time-deposit for ₹ 1,000, earning 6% compounded monthly.

(a) How much additional amount did the person invest on October 1?

(b) What was the maturity value of his time deposit on April 1 2012?

(c) How much total interest was earned?

Given that $(1 + i)^n$ is 1.03022500 for i=11/2 % n = 2 and $(1+ i)^n$ is 1.03037751 for i = 1/2 % and n = 6.

EFFECTIVE RATE OF INTEREST

- If interest is compounded more than once a year the effective interest rate for a year exceeds the per annum interest rate.
- Suppose you invest ₹ 10,000 for a year at the rate of 6% per annum compounded semi annually.
- Effective interest rate for a year will be more than 6% per annum since interest is being compounded more than once in a year.
- For computing effective rate of interest first we have to compute the interest. Let us compute the interest.

EFFECTIVE RATE OF INTEREST

- Interest for first six months = ₹ 10,000 × 6/100 × 6/12 = ₹ 300
- Principal for calculation of interest for next six months
- = Principal for first period one + Interest for first period
- = ₹ (10,000 + 300)
- = ₹ 10,300
- Interest for next six months = ₹ 10,300 × 6/100 × 6/12 = ₹ 309

EFFECTIVE RATE OF INTEREST

- Total interest earned during the current year
- = Interest for first six months + Interest for next six months
- = ₹ (300 + 309) = ₹ 609

EFFECTIVE RATE OF INTEREST

- We can compute effective rate of interest by following formula
- I = PEt
- Where I = Amount of interest
- E = Effective rate of interest in decimal
- t = Time period
- P = Principal amount

EFFECTIVE RATE OF INTEREST

- **Putting the values we have**
- **609 = 10,000 × E × 1**

$$E = \frac{609}{10{,}000}$$

$$= 0.0609 \text{ or}$$

$$= 6.09\%$$

EFFECTIVE RATE OF INTEREST

- If we compound the interest more than once a year effective interest rate for the year will be more than actual interest rate per annum.
- But if interest is compounded annually effective interest rate for the year will be equal to actual interest rate per annum.
- So effective interest rate can be defined as the equivalent annual rate of interest compounded annually if interest is compounded more than once in a year.

EFFECTIVE RATE OF INTEREST

The effective interest rate can be computed directly by following formula:

$$E = (1 + i)^n - 1$$

Where E is the effective interest rate

i = actual interest rate in decimal

n = number of conversion period

EFFECTIVE RATE OF INTEREST

Example 22: ₹ 5,000 is invested in a Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? What is effective rate of interest?

EFFECTIVE RATE OF INTEREST

Example 23: Find the amount of compound interest and effective rate of interest if an amount of ₹ 20,000 is deposited in a bank for one year at the rate of 8% per annum compounded semi annually.

EFFECTIVE RATE OF INTEREST

Example 24: Which is a better investment 3% per year compounded monthly or 3.2% per year simple interest? Given that $(1 + 0.0025)^{12} = 1.0304$.

EXERCISE- Set (B)

Choose the most appropriate option (a) (b) (c) or (d).

Que. 1 If P = ₹ 1,000, R = 5% p.a, n = 4; What is Amount and C.I. is

(a) ₹ 1,215.50, ₹ 215.50

(b) ₹ 1,125, ₹ 125

(c) ₹ 2,115, ₹ 115

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 2 ₹ 100 will become after 20 years at 5% p.a compound interest of

(a) ₹ 250

(b) ₹ 205

(c) ₹ 165.33

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 3 The effective rate of interest corresponding to a nominal rate 3% p.a payable half yearly is

(a) 3.2% p.a

(b) 3.25% p.a

(c) 3.0225% p.a

(d) none of these

DEPRECIATION

Depreciation is the fall in the value of an asset due to wear and tear , efflux of time , obsolescence .

$$A = P(1 - i)^n$$

Where ,

P = historical cost of asset
A = Scrap value / residual value
n = no . of periods
i = depreciation

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 4 **A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is**

(a) 4.5 years (appx.)

(b) 5.4 years (appx.)

(c) 5 years (appx.)

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 5 If A = ₹ 1,000, n = 2 years, R = 6% p.a compound interest payable half-yearly, then principal (P) is

(a) ₹ 888.50

(b) ₹ 885

(c) 800

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 6 The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

(a) 7 years

(b) 10 years

(c) 17 years (app)

(d) none of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 7 The difference between C.I and S.I on a certain sum of money invested for 3 years at 6% p.a is ₹ 110.16. The principle is

(a) ₹ 3,000

(b) ₹ 3,700

(c) ₹ 12,000

(d) ₹ 10,000

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 8 The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is

(a) ₹ 3,486.78

(b) ₹ 4,383

(c) ₹ 3,400

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 9 The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

(a) 7%

(b) 7.5%

(c) 5%

(d) 7.18%

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 10 The C.I on ₹ 16000 for 1 1/2 years at 10% p.a payable half -yearly is

(a) ₹ 2,222

(b) ₹ 2,522

(c) ₹ 2,500

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 11 The C.I on ₹ 40000 at 10% p.a for 1 year when the interest is payable quarterly is

(a) ₹ 4,000

(b) ₹ 4,100

(c) ₹ 4,152.51

(d) None of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 12 The difference between the S.I and the C.I on ₹ 2,400 for 2 years at 5% p.a is

(a) ₹ 5

(b) ₹ 10

(c) ₹ 16

(d) ₹ 6

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 13 The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

(a) 35 years.

(b) 30 years.

(c) 25 years

(d) none of these

COMPOUND INTEREST

Choose the most appropriate option (a) (b) (c) or (d).

Que. 14 The C.I on ₹ 4,000 for 6 months at 12% p.a payable quarterly is

(a) ₹ 243.60

(b) ₹ 240

(c) ₹ 243

(d) None of these

SINGLE CASH FLOW

If single amount is paid or received initially and then direct finally at the end .

Example : FD

ANNUITY

- In many cases you must have noted that your parents have to pay an equal amount of money regularly like every month or every year. For example payment of life insurance premium, rent of your house (if you stay in a rented house), payment of housing loan, vehicle loan etc. In all these cases they pay a constant amount of money regularly.
- Time period between two consecutive payments may be one month, one quarter or one year.

ANNUITY

- Sometimes some people received a fixed amount of money regularly like pension, rent of house etc.
- In all these cases annuity comes into the picture. When we pay (or receive) a fixed amount of money periodically over a specified time period we create an annuity.
- Thus annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.

ANNUITY

- To be called annuity a series of payments (or receipts) must have following features:

1. Amount paid (or received) must be constant over the period of annuity and
2. Time interval between two consecutive payments (or receipts) must be the same.

Can payments/receipts shown in the table for five years be called annuity?

Table - 4.1

Year end	Payments/Receipts (₹)
I	5,000
II	6,000
III	4,000
IV	5,000
V	7,000

Table - 4.2

Year end	Payments/Receipts (₹)
I	5,000
II	5,000
III	–
IV	5,000
V	5,000

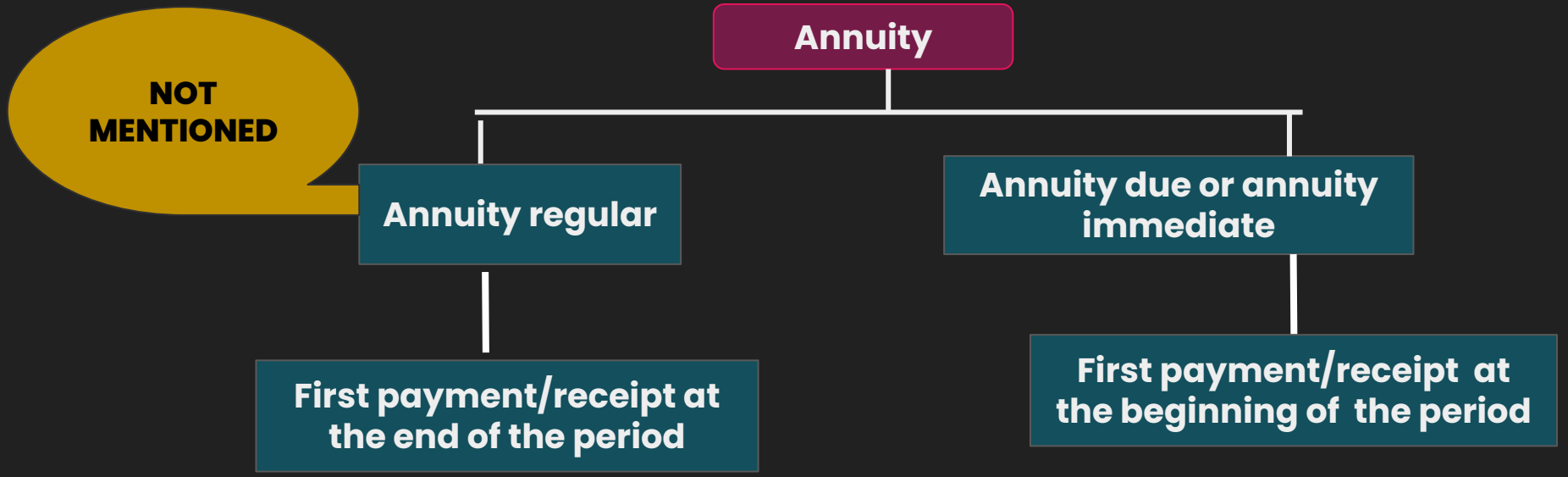
Table - 4.3

Year end	Payments/Receipts (₹)
I	5,000
II	5,000
III	5,000
IV	5,000
V	5,000

ANNUITY

- Payments/Receipts shown in table 4.1 cannot be called annuity. Payments/Receipts though have been made at regular intervals but amount paid are not constant over the period of five years.
- Payments/receipts shown in table 4.2 cannot also be called annuity. Though amounts paid/received are same in every year but time interval between different payments/receipts is not equal.
- Now consider table 4.3. You may note that all payments/receipts over the period of 5 years are constant and time interval between two consecutive payments/receipts is also same i.e. one year. Therefore payments/receipts as shown in table-4.3 can be called annuity.

TYPES ANNUITY



ANNUITY

1. ***Annuity regular:*** **In annuity regular first payment/receipt takes place at the end of first period. Consider following table:**

Table - 4.4

Year end	Payments/Receipts (₹)
I	5,000
II	5,000
III	5,000
IV	5,000
V	5,000

We can see that first payment/receipts takes place at the end of first year therefore it is an annuity regular.

ANNUITY

- ***Annuity Due or Annuity Immediate:*** **When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate. Consider following table:**

Table - 4.5

In the beginning of	Payment/Receipt (₹)
I year	5,000
II year	5,000
III year	5,000
IV year	5,000
V year	5,000

We can see that first receipt or payment is made in the beginning of the first year. This type of annuity is called annuity due or annuity immediate.

FUTURE VALUE – SINGLE CASH FLOW

- Future value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest.

- Suppose you invest ₹ 1,000 in a fixed deposit that pays you 7% per annum as interest. At the end of first year you will have ₹ 1,070. This consist of the original principal of ₹ 1,000 and the interest earned of ₹ 70. ₹ 1,070 is the future value of ₹ 1,000 invested for one year at 7%.

FUTURE VALUE – SINGLE CASH FLOW

- We know that
- $A_n = P(1 + i)^n$
- Where A = Accumulated amount
- n = number of conversion period
- i = rate of interest per conversion period in decimal
- P = principal

Future value of a single cash flow can be computed by above formula. Replace A by future value (F) and P by single cash flow (C.F.) therefore

$$F = C.F. (1 + i)^n$$

FUTURE VALUE – SINGLE CASH FLOW

Example 25: You invest ₹ 3000 in a two year investment that pays you 12% per annum. Calculate the future value of the investment.

PRESENT VALUE – SINGLE CASH FLOW

- Present value is today's value of tomorrow's money discounted at the interest rate.
- You invested ₹ 1000 at 7% and get ₹ 1,070 at the end of the year. If ₹ 1,070 is the future value of today's ₹ 1000 at 7% then ₹ 1,000 is present value of tomorrow's ₹ 1,070 at 7%.

PRESENT VALUE – SINGLE CASH FLOW

The present value P of the amount A_n due at the end of n period at the rate of i per interest period may be obtained by solving for P the below given equation

$$A_n = P(1 + i)^n$$

$$\text{i.e.} \quad P = \frac{A_n}{(1+i)^n}$$

PRESENT VALUE – SINGLE CASH FLOW

Example 29: What is the present value of ₹ 1 to be received after two years compounded annually at 10% interest rate?

PRESENT VALUE – SINGLE CASH FLOW

Example 30: Find the present value of ₹ 10,000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5 = 1.5386$.

FUTURE VALUE OF ANNUITY REGULAR

If A be the periodic payments, the future value A(n, i) of the annuity is given by

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Example 26: Find the future value of an annuity of ₹ 500 made annually for 7 years at interest rate of 14% compounded annually. Given that $(1.14)^7 = 2.5023$.

Example 27 : ₹ 200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$

FUTURE VALUE OF ANNUITY DUE

Future value of an Annuity due/Annuity immediate = Future value of annuity regular x (1+i)

where i is the interest rate in decimal.

Calculating the future value of the annuity due involves two steps.

Step-1 Calculate the future value as though it is an ordinary annuity.

Step-2 Multiply the result by (1+ i)

Example: Z invests ₹ 10,000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity. Given that $(1 + 0.08)^{10} = 2.15892500$.

SINKING FUND

- It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate.
- Interest is compounded at the end of every period.
- Size of the sinking fund deposit is same as future value of Annuity .

Example: How much amount is required to be invested every year so as to accumulate ₹ 300000 at the end of 10 years if interest is compounded annually at 10%?

PRESENT VALUE ANNUITY REGULAR

$$PVAR = A_1 \times \frac{1}{i}\left(1 - \frac{1}{(1+i)^n}\right)$$

Example 31: S borrows ₹ 5,00,000 to buy a house. If he pays equal instalments for 20 years and 10% interest on outstanding balance what will be the equal annual instalment?

Example 32: ₹ 5,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?

Example 33: Y bought a TV costing ₹ 13,000 by making a down payment of ₹ 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?

PRESENT VALUE ANNUITY DUE

Present value of annuity due/immediate for n years is the same as an annuity regular for (n-1)
years plus an initial receipt or payment in beginning of the period.
Calculating the present value
of annuity due involves two steps.

Step 1: Compute the present value of annuity as if it were a annuity regular for one period
short.

Step 2: Add initial cash payment/receipt to the step 1 value.

Example 34: Suppose your mom decides to gift you ₹ 10,000 every year starting from today for the next five years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

EXERCISE- Set (C)

Choose the most appropriate option (a) (b) (c) or (d).

Que. 1 The present value of an annuity of ₹ 3000 for 15 years at 4.5% p.a CI is

(a) ₹ 23,809.41

(b) ₹ 32,218.76

(c) ₹ 32,908.41

(d) none of these

EXERCISE- Set (C)

Que. 2 The amount of an annuity certain of ₹ 150 for 12 years at 3.5% p.a C.I is

(a) ₹ 2,190.28

(b) ₹ 1,290.28

(c) ₹ 2,180.28

(d) None of these

EXERCISE- Set (C)

Que. 3 A loan of ₹ 10,000 is to be paid back in 30 equal instalments. The amount of each installment to cover the principal and at 4% p.a CI is

(a) ₹ 587.87

(b) ₹ 587

(c) ₹ 578.30

(d) None of these

EXERCISE- Set (C)

Que. 4 A = ₹ 1,200 n = 12 years i = 0.08, V = ?

Using the formula $V = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$ value of v will be

(a) ₹ 3,039

(b) ₹ 3,990

(c) ₹ 9,930

(d) 9,043.30

EXERCISE- Set (C)

Que. 5 a = ₹ 100 n = 10, i = 5% find the FV of annuity

Using the formula FV = a / {1 + i) n – 1}, FV is equal to

(a) ₹ 1,258

(b) ₹ 2,581

(c) ₹ 1,528

(d) None of these

EXERCISE- Set (C)

Que. 6 If the amount of an annuity after 25 years at 5% p.a C.I is ₹ 50,000 the annuity will be

(a) ₹ 1,406.90

(b) ₹ 1,047.62

(c) ₹ 1,146.90

(d) None of these

EXERCISE- Set (C)

Que. 7 Given annuity of ₹ 100 amounts to ₹ 3137.12 at 4.5% p.a C. I. The number of years will be

(a) 25 years (appx.)

(b) 20 years (appx.)

(c) 22 years

(d) None of these

EXERCISE- Set (C)

Que. 8 A company borrows ₹ 10,000 on condition to repay it with compound interest at 5% p.a by annual installments of ₹ 1000 each. The number of years by which the debt will be clear is

(a) 14.2 years

(b) 10 years

(c) 12 years

(d) None of these

EXERCISE- Set (C)

Que. 9 Mr. X borrowed ₹ 5,120 at 12 1/2 % p.a C.I. At the end of 3 yrs, the money was repaid along with the interest accrued. The amount of interest paid by him is

(a) ₹ 2,100

(b) ₹ 2,170

(c) ₹ 2,000

(d) None of these

EXERCISE- Set (C)

Que. 10 Mr. Paul borrows ₹ 20,000 on condition to repay it with C.I. at 5% p.a in annual installments of ₹ 2000 each. The number of years for the debt to be paid off is

(a) 10 years

(b) 12 years

(c) 11 years

(d) 14.2 years

EXERCISE- Set (C)

Que. 11 A person invests ₹ 500 at the end of each year with a bank which pays interest at 10% p. a C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12^{th} time is.

(a) ₹ 11,761.36

(b) ₹ 10,000

(c) ₹ 12,000

(d) None of these

EXERCISE- Set (C)

Que. 12 The present value of annuity of ₹ 5,000 per annum for 12 years at 4% p.a C.I. annually is

(a) ₹ 46,000

(b) ₹ 46,850

(c) ₹ 15,000

(d) ₹ 46,925.40

EXERCISE- Set (C)

Que. 13 A person desires to create a fund to be invested at 10% CI per annum to provide for a prize of ₹ 300 every year. Using V = a/I find V and V will be

(a) ₹ 2,000

(b) ₹ 2,500

(c) ₹ 3,000

(d) None of these

EXERCISE- Set (D)

Que. 1 A = ₹ 5,200, R = 5% p.a., T = 6 years, P will be

(a) ₹ 2,000

(b) ₹ 3,880

(c) ₹ 3,000

(d) None of these

EXERCISE- Set (D)

Que. 2 If P = 1,000, n = 4 years., R = 5% p.a then C. I will be

(a) ₹ 215.50

(b) ₹ 210

(c) ₹ 220

(d) None of these

EXERCISE- Set (D)

Que. 3 The time in which a sum of money will be double at 5% p.a C.I is

(a) ₹ 10 years

(b) 12 years

(c) 14.2 years

(d) None of these

EXERCISE- Set (D)

Que. 4 If A = ₹ 10,000, n = 18yrs., R = 4% p.a C.I, P will be

(a) ₹ 4,000

(b) ₹ 4,900

(c) ₹ 4,500

(d) 4,936.30

EXERCISE- Set (D)

Que. 5 The time by which a sum of money would triple itself at 8% p. a C. I is

(a) 14.28 years

(b) 14 years

(c) 12 years

(d) None of these

EXERCISE- Set (D)

Que. 6 The present value of an annuity of ₹ 80 a years for 20 years at 5% p.a is

(a) ₹ 997 (appx.)

(b) ₹ 900

(c) ₹ 1,000

(d) None of these

EXERCISE- Set (D)

Que. 7 A person bought a house paying ₹ 20,000 cash down and ₹ 4,000 at the end of each year for 25 yrs. at 5% p.a. C.I. The cash down price is

(a) ₹ 75,000

(b) ₹ 76,000

(c) ₹ 76,375.50

(d) None of these

EXERCISE- Set (D)

Que. 8 A man purchased a house valued at ₹ 3,00,000. He paid ₹ 2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is

[Given log 10.6 = 1.0253 and log 31.19 = 1.494]

(a) ₹ 8,718.45

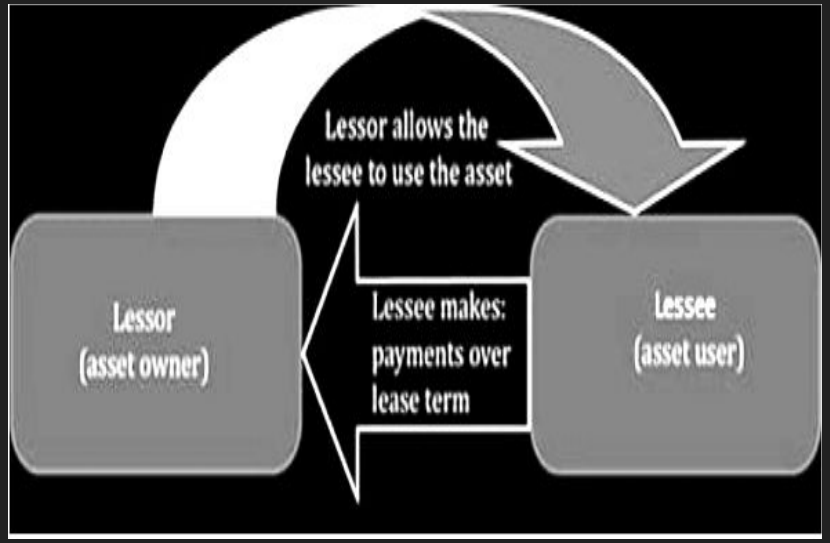
(b) ₹ 8,769.21

(c) ₹ 7,893.13

(d) None of these

LEASING

- **Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent.**



Present value of Annuity (lease rentals) are compared with asset cash down price to decide if leasing is preferable or not .

Example 36: ABC Ltd. wants to lease out an asset costing ₹ 3,60,000 for a five year period. It has fixed a rental of ₹ 1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

Solution First we have to compute the present value of the annuity of ₹ 1,05,000 for five years at the interest rate of 14% p.a. compounded annually.

The present value V of the annuity is given by

$V = A.P(n, i)$

$= 1,05,000 \times P(5, 0.14)$

$= 1,05,000 \times 3.43308 = ₹ 3,60,473.40$

which is greater than the initial cost of the asset and consequently leasing is favourable to the lessor.

Example 37: A company is considering proposal of purchasing a machine either by making full payment of ₹ 4,000 or by leasing it for four years at an annual rate of ₹ 1,250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

Solution The present value V of annuity is given by

$V = A.P(n, i)$

$= 1{,}250 \times P(4, 0.14)$

$= 1{,}250 \times 2.91371 = ₹\ 3{,}642.11$

which is less than the purchase price and consequently leasing is preferable.

CAPITAL EXPENDITURE (INVESTMENT DECISION)

- Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment.
- For taking investment decision we compare the present value of cash outflow and present value of cash inflows.
- If present value of cash inflows is greater than present value of cash outflows decision should be in the favour of investment

Example 38 : A machine can be purchased for ₹ 50000. Machine will contribute ₹ 12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not.

Solution The present value of annual contribution

$V = A.P(n, i)$

$= 12{,}000 \times P(5, 0.10)$

$= 12{,}000 \times 3.79079$

$= ₹\ 45{,}489.48$

which is less than the initial cost of the machine. Therefore machine must not be purchased.

Example 39 : A machine with useful life of seven years costs ₹ 10,000 while another machine with useful life of five years costs ₹ 8,000. The first machine saves labour expenses of ₹ 1,900 annually and the second one saves labour expenses of ₹ 2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

Solution The present value of annual cost savings for the first machine

= ₹ 1,900 × P (7, 0.10)

= ₹ 1,900 × 4.86842

= ₹ 9,249.99

= ₹ 9,250

Cost of machine being ₹ 10,000 it costs more by ₹ 750 than it saves in terms of labour cost.

The present value of annual cost savings of the second machine

= ₹ 2,200 × P(5, 0.10)

= ₹ 2,200 × 3.79079

= ₹ 8,339.74

Cost of the second machine being ₹ 8,000 effective savings in labour cost is ₹ 339.74. Hence the second machine is preferable.

VALUATION OF BOND

- A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.
- The bond issuer enters into contract with bondholder to pay interest.

Example 40: An investor intends purchasing a three year ₹ 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

PERPETUITY

- Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually .
- We can calculate PV of Perpetuity .
- FV of Perpetuity is not defined .

$$PVP = \frac{A_i}{i}$$

PVP = Present Value of Perpetuity

A_i = Installment (Annuity Value)

i = adjusted interest rate

Example 41: Ramesh wants to retire and receive ₹ 3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded monthly . How much will he need to set aside to achieve his perpetuity goal?

GROWING PERPETUITY

- A stream of cash flows that grows at a constant rate forever is known as growing perpetuity.

$$PVGP = \frac{A_i}{i - g}$$

PVGP = Present Value of Growing Perpetuity

A_i = Installment (Annuity Value)

i = adjusted interest rate

g = growth rate

Example 42: Assuming that the discount rate is 7% per annum, how much would you pay to receive ₹ 50, growing at 5%, annually, forever?

NET PRESENT VALUE

Net present value (NPV) = Present value of cash inflow – Present value of cash outflow

Decision Rule:

If NPV > 0 Accept the Proposal

If NPV < 0 Reject the Proposal

Example 43: Compute the net present value for a project with a net investment of ₹ 1,00,000 and net cash flows for year one is ₹ 55,000; for year two is ₹ 80,000 and for year three is ₹ 15,000. Further, the company's cost of capital is 10%?

[PVIF @ 10% for three years are 0.909, 0.826 and 0.751]

REAL RATE OF RETURN

- **Real Rate of Return:** The real interest rate is so named because it states the “real” rate that the lender or investor receives after inflation is factored in; that is, the interest rate that exceeds the inflation rate.
- *Real Rate of Return = Nominal Rate of Return – Inflation*

COMPOUND ANNUAL GROWTH RATE

Example: Suppose the revenues of a company for four years, V(t) in the above formula, have been

Year	2013	2014	2015	2016
Revenues	100	120	160	210

Calculate Compound annual Growth Rate.

Additional Questions Bank

Que. 1 The difference between compound and simple interest at 5% per annum for 4 years on ₹ 20,000 is ₹ _____

(a) 250

(b) 277

(c) 300

(d) 310

Que. 2 The compound interest on half-yearly rests on ₹ 10,000 the rate for the first and second years being 6% and for the third year 9% p.a. is ₹. _____.

(a) 2,200

(b) 2,287

(c) 2,285

(d) None

Que. 3 The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on yearly basis is ₹ _____.

(a) 9,070

(b) 9,000

(c) 9,061

(d) None

Que. 4 The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on half-yearly basis is ₹ _____.

(a) 9,070

(b) 9,069

(c) 9,061

(d) None

Que. 5 Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being 3.5%, how much each son receive after getting 25 years old?

(a) 50,000

(b) 51,994

(c) 52,000

(d) None

Que. 6 In how many years will a sum of money double at 5% p.a. compound interest?

(a) 15 years 3 months

(b) 14 years 2 months

(c) 14 years 3 months

(d) 15 years 2 months

Que. 7 In how many years a sum of money trebles at 5% p.a. compound interest payable on half-yearly basis?

(a) 18 years 7 months

(b) 18 years 6 months

(c) 18 years 8 months

(d) 22 years 3 months

Que. 8 A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹ 23,240 and ₹ 9,000 respectively. For how many years the machine was put to use?

(a) 7 years

(b) 8 years

(c) 9 years

(d) 10 years

Que. 9 A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000?

(a) 4 years 6 months

(b) 4 years 7 months

(c) 4 years 5 months

(d) 5 years 7 months approximately

Que. 10 A machine worth ₹ 4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%?

(a) 11 years 6 months

(b) 11 years 7 months

(c) 11 years 8 months

(d) 14 years 2 months approximately

Que. 11 Alibaba borrows ₹ 6 lakhs Housing Loan at 6% repayable in 20 annual installments commencing at the end of the first year. How much annual payment is necessary.

(a) ₹ 52,420

(b) ₹ 52,419

(c) ₹ 52,310

(d) ₹ 52,320

Que. 12 A sinking fund is created for redeeming debentures worth ₹ 5 lakhs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

(a) ₹ 12,006

(b) ₹ 12,040

(c) ₹ 12,039

(d) ₹ 12,035

Que. 13 A machine costs ₹ 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹ 25000. what amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?

(a) ₹ 16,000

(b) ₹ 16,500

(c) ₹ 16,050

(d) ₹ 16,005

Que. 14 Raja aged 40 wishes his wife Rani to have ` 40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. how much should he invest annually?

(a) 84,448

(b) 84,450

(c) 84,449

(d) 84,080

Que. 15 Appu retires at 60 years receiving a pension of 14,400 a year paid in half-yearly installments for rest of his life after reckoning his life expectation to be 13 years and that interest at 4% p.a. is payable half-yearly. What single sum is equivalent to his pension?

(a) 1,45,000

(b) 1,44,900

(c) 1,44,800

(d) 1,44,700